SECTION – A (MATHEMATICS)

PART - I

SINGLE OPTION CORRECT (+ 4, - 1, 0)

1. If $f_n = \sum_{k=0}^{n}(1+2^k)\binom{n}{k}$, then

(A) $f_{n+2} = 3f_{n+1} - 2f_n$ (B) $f_{n+2} = 3f_n + 2f_n$ (C) $f_{n+2} = 5f_{n+1} - 6f_n$ (D) $f_{n+2} = 6f_{n+1} - 5f_n$

2. Let $f(x) = \lim_{n\to\infty} \frac{(x^2+2x+3+\sin\pi x)^n - 1}{(x^2+2x+3+\sin\pi x)^n + 1}$, then

(A) $f(x)$ is continuous and differentiable for all $x \in R$.

(B) $f(x)$ is continuous but not differentiable for all $x \in R$.

(C) $f(x)$ is discontinuous at infinite number of points.

(D) $f(x)$ is discontinuous at finite number of points.

3. ABC is an isosceles triangle described in a circle of radius r. If AB = AC and h is the altitude from A to BC then $\lim_{h\to 0} \frac{\Delta}{p^3}$ equals to [where Δ is the area and p the is perimeter of the triangle ABC]

(A) 1/128r (B) 1/215r (C) 1/128 (D) 1/215

4. Let S be the set of all values of 'a' for which $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ is an invertible function and D is the set of domain of the function $g(x) = \sqrt{sin^{-1}(sin^{-1}x)}$, then S ∩ D equal to

(A) $[0, sin\ 1]$ (B) $[1, 4]$ (C) $[0,\ 4]$ (D) Null set

ROUGH SPACE

5. $\lim\limits_{x\to 1} \dfrac{\sqrt{x}+\sqrt{\sqrt{x}}+\sqrt{\sqrt{\sqrt{x}}+\sqrt{\sqrt{\sqrt{x}}}}-4}{x-1}$ is

(A) $\frac{1}{16}$ (B) $\frac{15}{16}$ (C) $\frac{7}{8}$ (D) $\frac{3}{4}$

6. The value of $\lim\limits_{x\to\infty}\left(\dfrac{a_1^{1/x}+a_2^{1/x}+\cdots+a_n^{1/x}}{n}\right)^{nx}, a_i > 0, i = 1,2,\ldots n,$ is

(A) $a_1 + a_2 + \cdots + a_n$ (B) $e^{a_1+a_2+\cdots+a_n}$ (C) $\frac{a_1+a_2+\cdots+a_n}{n}$ (D) $a_1\ a_2\ a_3 \ldots . a_n$

7. The equation of the tangent to the curve y = $\begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ at the origin is

(A) x = 0 (B) x = y (C) y = 0 (D) None of these

8. If $f(x) + f(x-4) = \sqrt{3}\, f(x-2)$ and f(0) = 10 , then number of solution of $\sin^{-1}(\sin x) = \frac{x}{\sum_{r=1}^{3} f(24r)}$

(A) 30 (B) 31 (C) 16 (D) 15

9. If $f(x) = \frac{1}{3}\left\{f(x+1) + \frac{5}{f(x+2)}\right\}$ and $f(x) > 0$ for all x ϵ R , then $\lim\limits_{x\to\infty} f(x)$ is

(A) $\sqrt{\frac{2}{5}}$ (B) $\sqrt{\frac{5}{2}}$ (C) ∞ (D) does not exist

10. If f(x) = $\begin{cases} (1+|\sin x|)^{\,a/|\sin x|}, & -\pi/6 < x < 0 \\ b & x = 0 \\ e^{\tan 2x/\tan 3x} & 0 < x < \pi/6 \end{cases}$ is a continuous function on $(-\pi/6, \pi/6)$, then -

(A) $a = 2/3, b = e^2$ (B) $a = 1/3, b = e^{1/3}$ (C) $a = 2/3, b = e^{2/3}$ (D) None of these

11. $f(x) = [x^2] - [x]^2$ is continuous at

(A) x = 0 (B) x = 1 (C) x = 2 (D) x = - 2

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12. Consider $f(x) = \left[\frac{2(\sin x - \sin^3 x)+|\sin x - \sin^3 x|}{2(\sin x - \sin^3 x)-|\sin x - \sin^3 x|}\right]$, $x \neq \frac{\pi}{2}$ for $x \in (0,\pi)$ $f\left(\frac{\pi}{2}\right) = 3$ where [.] = G.I.F. then

(A) $f(x)$ is continuous and differentiable at $x = \frac{\pi}{2}$

(B) $f(x)$ is continuous but not differentiable at $x = \frac{\pi}{2}$

(C) $f(x)$ is neither continuous nor differentiable at $x = \frac{\pi}{2}$

(D) none of these.

13. If $f'(3) = 2$ then $\lim_{h\to 0} \frac{f(3+h^2) - f(3-h^2)}{2h^2} =$

(A) 1 (B) 2 (C) 3 (D) 1/2

14. If $f(x) = [k + p \sin x], x \in (0,\pi)$, where $k \in I$, p is a prime number and [.] denotes integral part, then the number of points at which f is non – differentiable, is

(A) $p - 1$ (B) $p + 1$ (C) $2p - 1$ (D) $2p + 1$

15. Select the Wrong statements ([.] = GIF)

(A) $\left[0.\bar{9}\right] = 1$ (B) $\lim_{x\to 1^-}[x] = 0$

(C) $\lim_{x\to 0^+} x^{2x} = 0$ (D) $\lim_{x\to 0^+}\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+.....}}}} = 1$

16. Domain of $f(x) = \sin^{-1}(\cos^{-1}[\ln x]) + \sec^{-1}[x^2 + x + 1]$ (where [.] = G.I.F.)

(A) [0, e) (B) [1, e) (C) $(-\infty,-1] \cup [0,\infty)$ (D) $\left(-\infty,\frac{1}{e}\right] \cup [e,\infty]$

17. The derivative of $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & x \neq 1 \\ -\frac{1}{3}, & x = 1 \end{cases}$ at $x = 1$ is -

(A) $-\frac{2}{9}$ (B) $-\frac{2}{44}$ (C) 0 (D) None of these

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18. The function $f(x) = \begin{cases} (x+1)^{2-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \text{ is} \\ 0, & x = 0 \end{cases}$

(A) Discontinuous at only one point
(B) Discontinuous exactly at two points
(C) Continuous everywhere
(D) None of these

19. The value of $L = \lim_{x\to 0} \frac{\sin x + ae^{x} + be^{-x} + c\ln(1+x)}{x^{3}} = \text{Exist}$, is

(A) 1/2 (B) - 1/3 (C) - 1/6 (D) 3

20. If $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$, then

(A) Both f′ (0⁺) and f′ (0⁻) Do Not Exist
(B) f′ (0⁺) exist but f′ (0⁻) does not
(C) f′ (0⁺) = f′ (0⁻)
(D) None of these

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PART – II

Integer Type (+ 4, -1, 0).

21. $\lim_{x\to 0}\left(\left(1+\frac{1}{x}\right)^{x}+\left(\frac{1}{x}\right)^{x}+(\tan x)^{\frac{1}{x}}\right)$ is equal to _____

22. Suppose that the functions $F(x)$ and $G(x)$satisfy the following properties $F(3)=2, G(3)=4, G(0)=3, F'(3)=-1, G'(3)=0; G'(0)=2$ If $T(x)=F\big(G(x)\big)$ and $U(x)=\ln\big(F(x)\big)$, Then $|T'(0)+6U'(3)|$ has the value equal to _____

23. Consider $f(x)=x\sin\left([x]^4-5[x]^2+4\right)$, then number of points in (-5, 5) where $\lim_{x\to a} f(x)=$D.N.E. & $a\in(-5,5)$?

24. Let $f(x)=\begin{cases}\dfrac{\tan\left[x^2\right]\pi}{ax^2}+ax^3+b, & 0<x\le 1\\ 2\cos\pi x+\tan^{-1}x, & 1<x\le 2\end{cases}$. If f(x) is differentiable in (0, 2], then find the value of $2a+\pi-4b$.

25. Let $P_n=\left(1-\frac{1}{3}\right)^2\left(1-\frac{1}{6}\right)^2.....\left(1-\frac{2}{n(n+1)}\right)^2$ where $n\ge 2, n\in N$. If $\lim_{n\to\infty} P_n=\frac{a}{b}$ (where a & b are co-prime) then find the value of (a + b)

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26. Consider $f(x)=\begin{cases}\frac{1}{|x|}, & |x|\geq\frac{1}{2}\\ a+bx^2, & |x|<\frac{1}{2}\end{cases}$. If f(x) is differentiable at x = 1/2, then find the value of a – b

27. If $\lim\limits_{x\to0}\frac{ax+b-\sqrt{4+\sin x}}{\tan x}=\frac{27}{4}$, where a and b are constants. The value of (a - b) is _____

28. If $L=\lim\limits_{x\to0}\frac{(\sin x-x)^2+1-\cos(x^3)}{x^5\sin x}$ then $\frac{72L}{19}$ is equal to _____

29. Let $f(x)=\begin{cases}\frac{\alpha\cot x}{x}+\frac{\beta}{x^2}, & 0<|x|\leq1\\ \frac{1}{3}, & x=0\end{cases}$. If f(x) is continuous at x = 0, then find the value of $\alpha^2+\beta^2$.

30. If $f(x)=\begin{cases}\lambda+\tan[x], & x>0\\ 2, & x=0\\ \mu+\left[\frac{\tan x}{x}\right], & x<0\end{cases}$, where [.] = GIF. If f(x) is continuous at x = 0, then $(\lambda+\mu)$ is equal to __

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ANSWER KEY

1. C	2. A	3. C	4. D
5. B	6. D	7. C	8. B
9. B	10. C	11. B	12. A
13. B	14. C	15. C	16. A
17. A	18. A	19. B	20. A
21. 2	22. 5	23. 6	24. 9
25. 10	26. 7	27. 5	28. 2
29. 2	30. 3